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Infinite Elements for Three-Dimensional Fluid-Structure Interaction Problems

by

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This report describes a method for using three-dimensional infinite elements to compute acoustic fields in the infinite region containing water that surrounds a submerged structure. In this method, finite elements are used to represent a bounded region that contains the structure and in some cases contains part of the surrounding water. The unbounded region that surrounds the bounded region represented by finite elements and that contains the remainder of the water is represented with infinite elements. The infinite elements implement the added mass approximation to the Helmholtz equation.

Several other methods are available for modeling the effects of the surrounding water. For the added mass approximation, an alternative to using infinite elements is to model parts of the surrounding water with several layers of finite elements. To determine whether the method using infinite elements gives an advantage, the efficiency of that method is compared with the efficiency of the alternative method. The relative efficiencies are determined by comparing, for each method, the effort to construct the model and the accuracy of the solutions obtained. Evaluation of the infinite elements shows that they produce accurate results especially if they are used over a layer of finite fluid elements. However, when solutions

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ABSTRACT

This report describes a method for using three-dimensional infinite elements to compute acoustic fields in the infinite region containing water that surrounds a submerged structure. In this method, finite elements are used to represent a bounded region that contains the structure and in some cases contains part of the surrounding water. The unbounded region that surrounds the bounded region represented by finite elements and that contains the remainder of the water is represented with infinite elements. The infinite elements implement the added mass approximation to the Helmholtz equation.

Several other methods are available for modeling the the effects of the surrounding water. For the added mass approximation, an alternative to using infinite elements is to model part of the surrounding water with several layers of finite elements. To determine whether the method using infinite elements gives an advantage, the efficiency of that method is compared with the efficiency of the alternative method. The relative efficiencies are determined by comparing, for each method, the effort to construct the model and the accuracy of the solutions obtained. Evaluation of the infinite elements shows that they produce accurate results especially if they are used over a layer of finite fluid elements. However, when solutions obtained using infinite elements are compared with solutions obtained using only finite elements, it is seen that, for added mass problems, infinite elements do not offer a significant advantage over finite elements. Infinite elements are expected to provide a greater advantage when used to solve the Helmholtz equation.

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INTRODUCTION

To compute magnetic or acoustic fields about submerged marine structures, the structure, the surrounding water, and interactions between them must be modeled. The field and the structure can each be represented by finite elements and the representations can be coupled using a method developed by Zienkiewicz and Newton.[1] If the structure is submerged in the sea, the extent of the water surrounding it is so great that the region containing the water is best represented as having infinite extent. If finite elements are used to solve such a problem, one can model the structure and a finite part of the infinite region surrounding the structure. This leaves the problem of accounting for the remainder of the surrounding infinite region.

One method of accounting for the infinite region of water surrounding a submerged structure is to represent the region with infinite elements. To solve problems involving sound radiated or scattered by a submerged structure, the water is treated as an acoustic fluid, that is, a fluid having an acoustic pressure field described by the wave equation. For low frequencies, an approximation called the added mass approximation (described later in this report) simplifies the solution of the acoustic problems. This report documents the development of infinite elements and the evaluation of these elements for fluid-structure interaction probler. involving the vibrations of submerged structures in which the added mass approximation is made.

To be useful, an infinite element must not only be able to produce solutions with good accuracy but must achieve this accuracy while requiring less effort for modeling and computation than competing methods. Several competing methods of accounting for the surrounding infinite region are as follows:

- The surrounding region is truncated by modeling a large part of it with finite elements and applying boundary conditions that approximate the effects of the remainder of the infinite region.
- The solution is expanded as a series of analytic functions in the surrounding region with the coefficients of the series terms introduced as unknowns.
- The effects of the surrounding region are represented by an integral equation on the boundary of the region surrounding the finite elements.

Zienkiewicz et al.[2] compare modeling with these methods and with infinite elements and find that each has certain advantages and disadvantages.

The advantages of using infinite elements are that they maintain symmetry and reasonable bandwidths in the matrices produced, and the method is easy to implement using available finite element programs. On the other hand, some formulations of infinite elements introduce more degrees of freedom than do competing methods, and in some cases an extra effort must be made to ensure that element shapes and modeling configurations provide unique mappings. See Bettess and Bettess[3] for a discussion on avoiding nonunique mappings. For some problems, these characteristics may require additional effort in producing the numerical model.

In this report the surrounding infinite medium is assumed to be an acoustic fluid and infinite elements are used to represent this fluid. An infinite element represents a sector of the infinite region extending from the boundary of the finite region. In both finite and infinite elements the unknown function is approximated by shape functions and a functional involving the shape functions is integrated over the area or volume of the element. To obtain convergent integrals for infinite elements, one of two schemes is used.[3] The first incorporates decay factors in the shape functions that vary in the direction that extends to infinity, the second scheme maps the infinite element into a standard square or cube. In the second scheme, the decay factors appear in the mappings that compress the infinite element into a finite region. In either case, the decay factors may take one of several forms, they may decrease exponentially with the distance r from a fixed point, or decrease as a power of r. Many earlier developments of infinite elements used exponential decay factors,[4] but these require the choice of a somewhat arbitrary parameter called the decay length, and at moderate radii the shape functions do not approximate the behavior of solutions as well as the reciprocal of a power of the radius. Later work in infinite elements[5] has emphasized decay factors that decrease as $1/r^n$, and these factors are used in this work.

Several beginnings of applications of infinite elements to vibrations of submerged structures have been made in which promising results have been reported.[6,7] The infinite elements used in these papers contained exponential decay factors, and the papers do not report useful comparisons with alternative methods of computation.

In problems with steady state solutions for the acoustic response of submerged structures, the behavior of the surrounding acoustic fluid is described by the Helmholtz equation. For low frequencies, an approximation can be made in which the effect of the surrounding fluid becomes that of increasing the mass of the structure. This approximation is called the added mass approximation. The results given in this report show that, for fluid-structure interaction problems using the added mass approximation, infinite elements give good results and in fact are competitive with an alternative method. However, in the case of the added mass approximation the infinite elements do not offer a clear advantage over the alternative. This conclusion does not apply to the application of infinite elements to Helmholtz equation problems in which infinite elements are expected to offer advantages over their competitors, since for these problems the acoustic wave characteristics of the sound field are important and the infinite elements will reflect these characteristics.

FINITE AND INFINITE ELEMENT MODEL OF A FLUID-STRUCTURE SYSTEM

A finite element model is used to compute the natural frequencies of a structure submerged in an acoustic fluid or the structure's response to harmonic excitation over a range of frequencies. The finite element model of the acoustic fluid in which a structure is submerged may contain both finite and infinite fluid elements. Both finite and infinite fluid elements may be of various orders with various numbers of nodes. The infinite and finite fluid elements used in this report are brick-type three-dimensional elements with eight nodes.

Paradoxically, an infinite element is a type of finite element. (The adjective "finite" indicates that a finite element has *at least finite* dimensions, that it is not an infinitesimal element as used in the derivation of equations of physics.) Thus infinite elements inherit many of the desirable features of finite elements. The configuration of the model in terms of nodes and connections and the form of the resulting matrices do not change when infinite elements are included. Computing the matrix entries contributed by infinite elements follows the same steps as computing matrix entries for finite elements, although some special considerations are needed to ensure that integrals over infinite regions converge. The contribution of an infinite element is included in a finite element matrix in the same manner as the contribution of a finite element. The finite element matrices remain symmetric and banded if infinite elements are included. These properties make it easy to include infinite elements in existing finite element codes, and in the following discussions we will use expressions such as finite element method, finite element model, or finite element matrix, even though infinite elements may be included.

To use infinite elements to model a structure submerged in an unbounded acoustic fluid such as the sea, one models a finite region containing the structure and optionally part of the fluid close to the structure with finite elements and the remainder with infinite elements. A model of the surrounding fluid that contains only finite fluid elements consists of one or more layers of finite elements covering the surface formed by the interface between the structure and the fluid. If infinite fluid elements are used, one or more outer layers of finite elements are replaced by a layer of infinite elements. Each infinite element represents a sector of the infinite region radiating outward from the boundary of the region modeled with finite elements. Figure 1 shows the three configurations of finite and infinite elements modeling the surrounding fluid. The most advantageous arrangement in terms of reducing modeling and computation costs is that all the finite fluid elements are replaced so that only infinite elements remain to model the acoustic fluid.

The finite element method produces one matrix equation for the displacements in the structure and another matrix equation for the acoustic pressure field in the surrounding fluid. These matrix equations are coupled by terms arising from interactions between the fluid and the structure. To compute natural frequencies of the fluid-structure system, one assumes a time harmonic solution, and the matrix equation becomes the equation for an eigenvalue problem that determines the frequencies. To compute the response to a harmonic exciting force, one solves a system of equations with complex coefficients and a sinusoidal forcing term for the nodal displacements.

Standard finite element modeling procedures produce the matrix equation for the vector of nodal displacements u in the structure[8]

$$M\ddot{u} + B\dot{u} + Ku = f$$

in which M, B, and K are the structural mass, damping, and stiffness matricies and f is the vector of forces acting on the nodes of the structure.

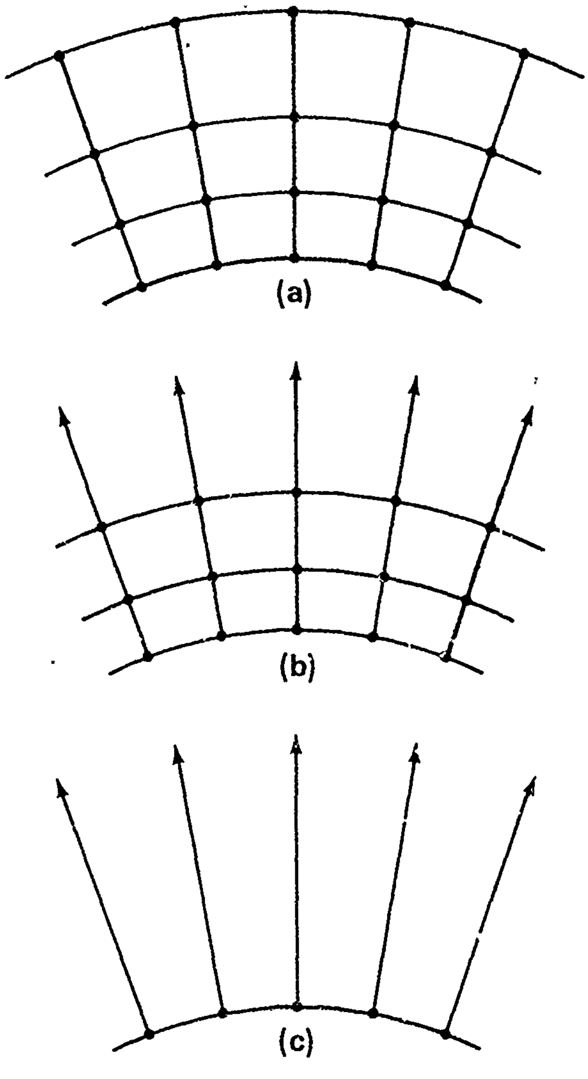

Fig. 1. Infinite elements and finite elements modeling the surrounding fluid: (a) Fluid modeled with three layers of finite elements, (b) Fluid modeled with infinite elements over two layers of finite elements, and (c) Fluid modeled with only infinite elements.

The differential equation for the acoustic pressure P in the fluid is the wave equation

$$\frac{1}{c^2}\frac{\partial^2 P}{\partial t^2} - \nabla^2 P = 0$$

where c is the speed of sound in the fluid. A method of analogies that uses solid three-dimensional structural elements produces a matrix equation for the pressure field in the acoustic fluid. The method of analogies[9] is applied by giving Young's modulus $E = 1.0$ and the shear modulus $G = 10^5$ and produces a matrix equation for the pressure field in the acoustic fluid

$$Q\ddot{p} + Hp = 0$$

where p is a vector of pressures at the nodes of the fluid finite elements.

If the acoustic pressure is harmonic in time with frequency ω, that is, $P(x,t) = p(x)e^{i\omega t}$ where x is a point in space and $k = \omega/c$, the wave equation becomes the Helmholtz equation

$$k^2p + \nabla^2 p = 0. \tag{1}$$

Interactions at the fluid-structure interface couple the structure and fluid matrix equations See Schroeder and Marcus[10] and Everstine et al.[11] for details of the analysis of the fluid-structure coupling The acoustic pressure acting on the interface produces a force vector (with the convention that forces acting outward from the surface are positive), so the matrix equation for the structure becomes

$$M\ddot{u} + B\dot{u} + Ku = -Ap + F$$

where the matrices M, B, and K are defined previously, A is a matrix whose entries reflect the areas of elements on the interface, and F is the vector of forces other than the acoustic forces on the nodes of the structure. The motion of the structure also affects the acoustic pressure field in the fluid. The normal component $\ddot{u}_{\hat{n}}$ of the acceleration of the interface between the structure and fluid is related to the gradient of the acoustic pressure field by the equation

$$\frac{\partial p}{\partial \hat{n}} = -\rho \ddot{u}_{\hat{n}}$$

where ρ is the density of the fluid and $\partial/\partial\hat{n}$ is the derivative in the direction $\hat{n}$ normal to the interface This relation is added as a boundary condition and produces a force-type term in the equation for the acoustic pressure field. In the fluid finite element matrix equation, this term is an area matrix term

$$Q\ddot{p} + Hp = -\rho A^T.$$

For the time harmonic case this equation becomes

$$k^2\,Qp + Hp = -\rho A^T. \tag{2}$$

The matrix A is the same area matrix that appears in the matrix equation for the structure. Either a consistent or a lumped formulation may be chosen to compute the matrix A. For problems using the added mass approximation, the lumped formulation has worked well[10,11] and was used in this work.

Combining the matrix equations for the structure and the acoustic field produces a system of equations for the coupled fluid-structure problem

$$\begin{aligned} M\ddot{u} + B\dot{u} + Ku + Ap &= F \\ -\rho A^T\ddot{u} + Q\ddot{p} \qquad\quad + Hp &= 0. \end{aligned} \tag{3}$$

These equations are written in the form of a matrix equation

$$\begin{bmatrix} M & 0 \\ -\rho A_T & Q \end{bmatrix}\begin{pmatrix} \ddot{u} \\ \ddot{p} \end{pmatrix} + \begin{bmatrix} B & 0 \\ 0 & 0 \end{bmatrix}\begin{pmatrix} \dot{u} \\ \dot{p} \end{pmatrix} + \begin{bmatrix} K & A \\ 0 & H \end{bmatrix}\begin{pmatrix} u \\ p \end{pmatrix} = \begin{pmatrix} F \\ 0 \end{pmatrix}.$$

This equation can be made symmetric by defining a new variable, essentially a velocity potential[12]

$$q(t) = \int_0^t p(\tau)d\tau,$$

and integrating the second row to obtain the matrix equation in the form

$$\begin{bmatrix} M & 0 \\ 0 & Q \end{bmatrix}\begin{pmatrix} \ddot{u} \\ \ddot{q} \end{pmatrix} + \begin{bmatrix} B & A \\ -A_T & 0 \end{bmatrix}\begin{pmatrix} \dot{u} \\ \dot{q} \end{pmatrix} + \begin{bmatrix} K & 0 \\ 0 & H \end{bmatrix}\begin{pmatrix} u \\ q \end{pmatrix} = \begin{pmatrix} F \\ 0 \end{pmatrix}.$$

If the equation is harmonic in time (this is the situation to which the Helmholtz equation applies), then the coupled matrix equation becomes

$$\left\{-\omega^2\begin{bmatrix} M & 0 \\ 0 & Q \end{bmatrix} + i\omega\begin{bmatrix} B & A \\ -A_T & 0 \end{bmatrix} + \begin{bmatrix} K & 0 \\ 0 & H \end{bmatrix}\right\}\begin{pmatrix} u \\ q \end{pmatrix} = \begin{pmatrix} F \\ 0 \end{pmatrix}.$$

The response to a harmonic excitation of frequency ω is computed by solving the preceding equation for displacements at selected grid points resulting from an applied harmonic load $F(t) = e^{i\omega t}$ applied at a load point. For natural frequencies, the eigenvalue equation

$$\omega^2 \begin{bmatrix} M & 0 \\ 0 & Q \end{bmatrix} + \omega \begin{bmatrix} B & A \\ -A_T & 0 \end{bmatrix} + \begin{bmatrix} K & 0 \\ 0 & H \end{bmatrix} = 0$$

is solved for the complex frequencies ω.

In this report the added mass approximation, valid for low frequencies, is used. For small ω the Helmholtz equation (Eq. 1) is approximated by $\nabla^2 p = 0$, the fluid matrix equation (Eq. 2) becomes $Hp = -\rho A^T$; and the coupled fluid-structure system of equations (Eq. 3) is

$$\begin{aligned} M\ddot{u} + B\dot{u} + Ku + Ap &= 0 \\ -\rho A^T \ddot{u} \qquad\qquad + Hp &= 0. \end{aligned}$$

If the second equation is solved for p and this expression for p is substituted into the first equation, the first equation will include the term $\rho A H^{-1} A^T \ddot{u}$, which has the form of a mass term and is called the added mass. This substitution was used to compute the natural frequencies of a submerged plate. For computational reasons, the substitution was not used explicitly for the frequency response of a submerged cylinder.

INFINITE ELEMENT FORMULATION

The infinite elements used in this study are of the mapped reciprocal decay type.[5] They are three-dimensional brick-type elements with eight nodes. The formulation of these elements is similar to that of an isoparametric finite element in that the region represented by the element is mapped into a standard finite cube. The mapping for an infinite element incorporates decay factors, factors that decrease to zero fast enough to map the infinite region into a standard finite cube. The decay factors used in these elements are reciprocals of powers of the distance r from a convenient origin to the mapped point.

The mapping function is

$$
\begin{aligned}
x = {} & x_1F_2(\xi)F_2(\eta)H_1(\varsigma) + x_2F_1(\xi)F_2(\eta)H_1(\varsigma) \\
& + x_3F_1(\xi)F_1(\eta)H_1(\varsigma) + x_4F_2(\xi)F_1(\eta)H_1(\varsigma) \\
& + x_5F_2(\xi)F_2(\eta)H_2(\varsigma) + x_6F_1(\xi)F_2(\eta)H_2(\varsigma) \\
& + x_7F_1(\xi)F_1(\eta)H_2(\varsigma) + x_8F_2(\xi)F_1(\eta)H_2(\varsigma)
\end{aligned}
$$

and the shape functions are

$$
\begin{aligned}
N_1(\xi,\eta,\varsigma) &= F_2(\xi)F_2(\eta)G_1(\varsigma) \\
N_2(\xi,\eta,\varsigma) &= F_1(\xi)F_2(\eta)G_1(\varsigma) \\
N_3(\xi,\eta,\varsigma) &= F_1(\xi)F_1(\eta)G_1(\varsigma) \\
N_4(\xi,\eta,\varsigma) &= F_2(\xi)F_1(\eta)G_1(\varsigma) \\
N_5(\xi,\eta,\varsigma) &= F_2(\xi)F_2(\eta)G_2(\varsigma) \\
N_6(\xi,\eta,\varsigma) &= F_1(\xi)F_2(\eta)G_2(\varsigma) \\
N_7(\xi,\eta,\varsigma) &= F_1(\xi)F_1(\eta)G_2(\varsigma) \\
N_8(\xi,\eta,\varsigma) &= F_2(\xi)F_1(\eta)G_2(\varsigma)
\end{aligned}
$$

where the functions F, G, and H are defined by

$$
\begin{aligned}
F_1(\tau) &= (1-\tau)/2; & F_2(\tau) &= (1+\tau)/2; \\
G_1(\varsigma) &= \varsigma(\varsigma-1)/2; & G_2(\varsigma) &= 1-\varsigma^2; \\
H_1(\varsigma) &= -2\varsigma/(1-\varsigma); & H_2(\varsigma) &= (1+\varsigma)/(1-\varsigma).
\end{aligned}
$$

In the preceding definitions, a point in space is $x = xi + yj + zk$, for **i**, **j**, and **k** unit vectors in the x-, y-, and z-directions, and x_i and N_i are the locations and shape functions for the ith node in an infinite element. Using these functions, one can compute the Jacobian of the mapping function and the gradients of the shape functions.

Then the integrals over an element

$$\int\int\int N_iN_j \, dxdydz = \int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1} N_iN_j \, |J| \, d\xi d\eta d\varsigma$$

$$\int\int\int \nabla N_i \nabla N_j \, dxdydz = \int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1} \nabla N_i \nabla N_j \, |J| \, d\xi d\eta d\varsigma$$

are computed by numerical quadrature over the finite region of the standard cube. These values are the contributions of the infinite elements to the finite element matrices. Figure 2 shows the mapping and the standard cube.

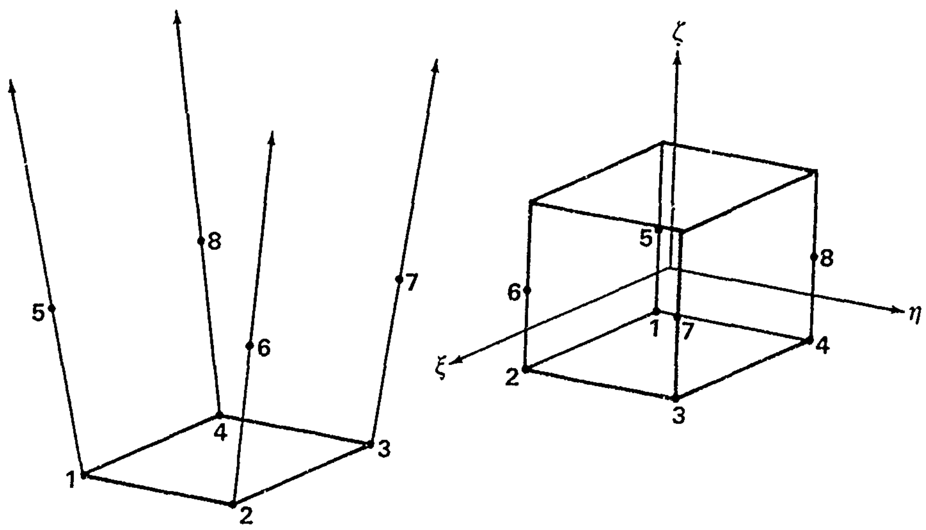

Fig. 2. An infinite element and its parent cube.

The standard cube in the ξ, η, ς space is defined by $-1 \leq \xi,\eta,\varsigma \leq 1$. The variable ς is mapped into the direction going to infinity in the infinite element. The face defined by nodes 1-4 of the infinite element lies on the outer surface of the region modeled by finite elements. This face corresponds to the face of the standard cube that is defined by $\varsigma = -1$. Each of the pairs of nodes 1,5, 2,6, 3,7, and 4,8 of the infinite element determines one of the four rays that determine the region represented by the element. Nodes 5-8 of the infinite element correspond to the four nodes in the standard cube at which $\varsigma = 0$. Since the infinite element is mapped into a finite region, a standard Gaussian quadrature is used to compute the preceding integrals.

EVALUATION OF THE INFINITE ELEMENTS

The infinite elements developed for use with the added mass approximation were evaluated by performance tests in which the accuracy of solutions obtained using infinite elements was compared with the accuracy obtained using only finite elements. These tests of the infinite elements are intended not only to show whether the elements produce accurate solutions, but also to determine whether they offer an advantage over other methods, that is, whether they produce as good or better accuracy with less modeling effort or less computer time. Although both finite and infinite elements can be formulated with higher order mapping and shape functions, both were chosen to have eight nodes and to have shape functions that are linear in the plane of the fluid-structure interface. The infinite elements with eight nodes have shape functions that are quadratic in the infinite direction. For fluid-structure interaction problems using the added mass approximation, it has been shown that good results can be obtained using only a few layers of finite elements to model the surrounding fluid.[13] Therefore, for the infinite elements to be competitive with finite elements, comparable results should be obtained using infinite elements only, or much better results should be obtained by using infinite elements with one layer of finite elements.

The computations for carrying out the evaluations of the infinite elements were made using the finite element program NASTRAN. Separate programs generate the matrices for the infinite elements, and NASTRAN reads the matrix entries during execution.

As a preliminary test to validate the formulation of the infinite elements, the elements were used to compute the solution to a Dirichlet problem $\nabla^2 u = 0$ outside a boundary formed by a cylindrical shell with finite length and closed ends. The Laplace equation describes the acoustic field in the added mass approximation. This test evaluates the performance of the infinite elements for representing the Laplace operator while avoiding the complications of the fluid-structure interaction problem. Solutions obtained using infinite elements were compared with solutions obtained using only finite elements. The standard for the comparisons was a solution obtained by the boundary element program BEASY. Details of the evaluation and results of this test are given in Appendix A. The results of this test show that, when modeling the exterior region for the three-dimensional Laplace equation, the use of infinite elements produces more accurate solutions than the use of finite elements.

A factor in determining the relative efficiencies of two methods is the expense of setting up and solving a problem using each method. Since the infinite and finite elements used in this work each contain eight nodes, the effort needed to produce a finite element model and to perform the computations using a layer of infinite elements covering the surface of the structure is approximately equal to the effort needed for one layer of finite elements. Thus the effort required in setting up and solving a fluid model consisting of two layers of finite fluid elements is about the same as the effort required for a model consisting of infinite fluid elements over a layer of finite fluid elements.

Two problems were used to evaluate the performance of the infinite elements in applications to fluid-structure interaction problems.

The first problem is to compute some lower natural frequencies of a submerged flat plate. The plate used in this test had the same dimensions and material properties as one which Marcus[13] used to demonstrate the application of finite elements for computing natural frequencies of submerged plates. He showed that only a few layers of finite fluid elements are sufficient to compute accurately the natural frequencies of submerged plates, and he obtained good accuracy using three layers. Therefore, for this test, a set of standard natural frequencies was obtained using a conservative model of the surrounding fluid consisting of five layers of finite fluid elements. The accuracy of the natural frequencies computed for the test cases was determined by comparing those frequencies with the set of standard natural frequencies. Three models of the surrounding fluid were used for computing the natural frequencies of the plate. The first model, the most desirable from the standpoint of ease of modeling and economy in computation, represents the surrounding fluid with only infinite elements. The second model uses infinite elements to represent the infinite region of fluid and a layer of finite fluid elements under the infinite elements to obtain better resolution of the sound field in the fluid adjacent to the structure. To get this improved resolution, one must invest additional effort in preparing the finite element model and in computing the solution. The third model of the surrounding fluid consists of two layers of finite fluid elements and no infinite elements. This model includes the same number of elements to be constructed and (except for some boundary constraints) the same number of degrees of freedom as the second model. Thus the third model requires nearly the same investment in modeling effort and computational expense as the second. Appendix B gives details of the model used and techniques of the computation.

Table 1 gives the frequencies computed in these tests.

TABLE 1. Natural frequencies of a submerged plate.

Layers of Finite Elements	Infinite Elements Used	Frequencies(normalized)				
		Mode				
		1	2	3	4	5
5	No	1.000	1.000	1.000	1.000	1.000
0	Yes	1.044	1.024	1.033	1.016	1.020
1.	Yes	1.024	1.024	1.021	1.006	1.015
2	No	1.052	1.024	1.036	1.027	1.036

In this table, the natural frequencies have been normalized by dividing the frequencies for each mode by the standard frequency for that mode. (The unnormalized frequencies are tabulated in Appendix B) Table 1 shows that using only infinite elements to represent the surrounding fluid produces good results and that adding a layer of finite elements under the infinite elements produces better results. However, it also shows that using infinite elements does not give a significant advantage over using only finite elements, since using a layer of finite elements and infinite elements requires approximately the same modeling and computational effort as using two layers of finite elements.

The second problem for evaluating the performance of the infinite elements is to compute the frequency response of a submerged cylindrical shell. The cylindrical shell had a finite length and was subjected to harmonic excitation through a range of frequencies. It was driven at one point by a sinusoidal force of amplitude 1.0 and the response was computed at a separate transfer point. Appendix B gives details of this test. As in the preceding problem, the standard for comparison is the solution obtained when the fluid was modeled using five layers of finite fluid elements. Figures 3 to 6 show comparisons of responses obtained using infinite fluid elements alone or over a layer of finite fluid elements with the responses obtained using only one or two layers of finite fluid elements. The response shown in these figures is impedance. The impedance is given in units of lb-sec/in and equals the force applied at the drive point divided by the velocity of the transfer point, the point at which the response is computed. Figures 2 and 3 show that the use of only infinite fluid elements does not yield results as accurate as does the use of either one or two layers of finite fluid elements. Figure 4 shows that the results obtained using infinite fluid elements over a layer of finite fluid elements are accurate and are an improvement over the results produced using one layer of finite fluid elements. Figure 5, however, shows that the same configuration using infinite fluid elements produces no more accurate results than does using an equally costly model that contains two layers of finite fluid elements.

DISCUSSION OF TEST RESULTS

The evaluations of the infinite elements show that they produce accurate results, especially if they are used over a layer of finite fluid elements. This conclusion is consistent with earlier work on two-dimensional infinite elements in which good accuracy was obtained when finite fluid elements were used with infinite elements.[14] However, when solutions obtained using infinite elements are compared with solutions obtained using only finite elements, taking into consideration both the effort to construct the model and the accuracy of the solutions, it is seen that for added mass problems infinite elements do not offer an advantage over finite elements. For a more complete evaluation of the infinite elements, elements with higher order mapping and shape functions could be tested. But for a fair comparison, a higher order infinite element must compete with an equally higher order finite element so that, as the accuracy of the infinite elements improves, so also will the accuracy of its competitor. It was judged that, for added mass problems, the prospects that the accuracy of the infinite elements would improve more than that of the competing method was not good enough to warrant the investment of additional study, and there are no plans to continue the investigation of infinite elements for added mass problems.

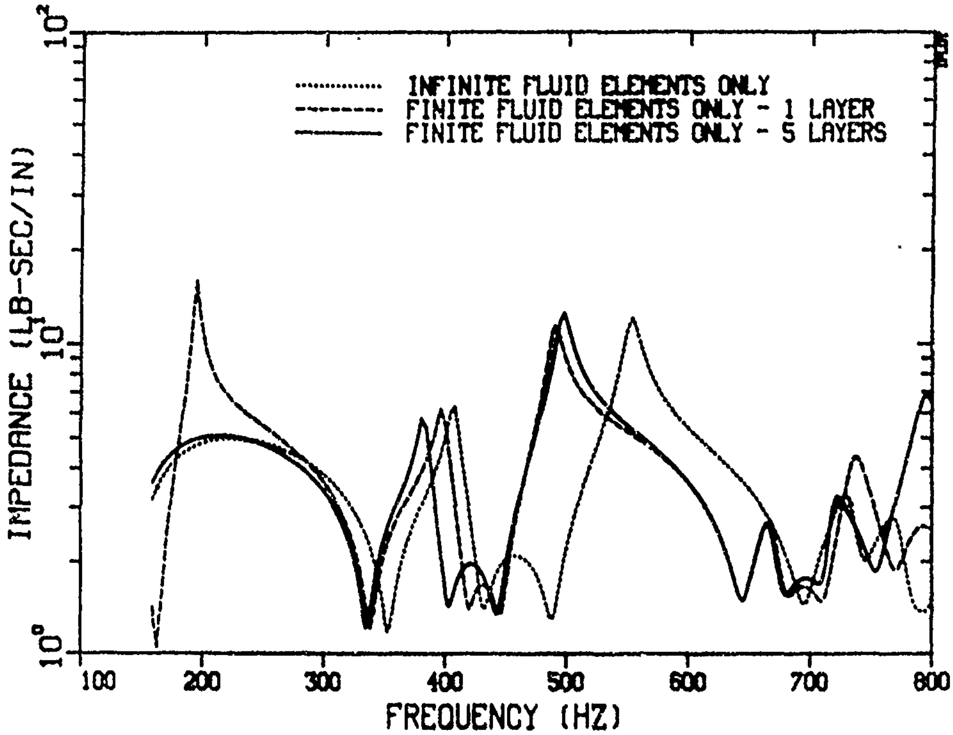

Fig. 3. Frequency response, comparisons using infinite elements only with one layer of finite elements.

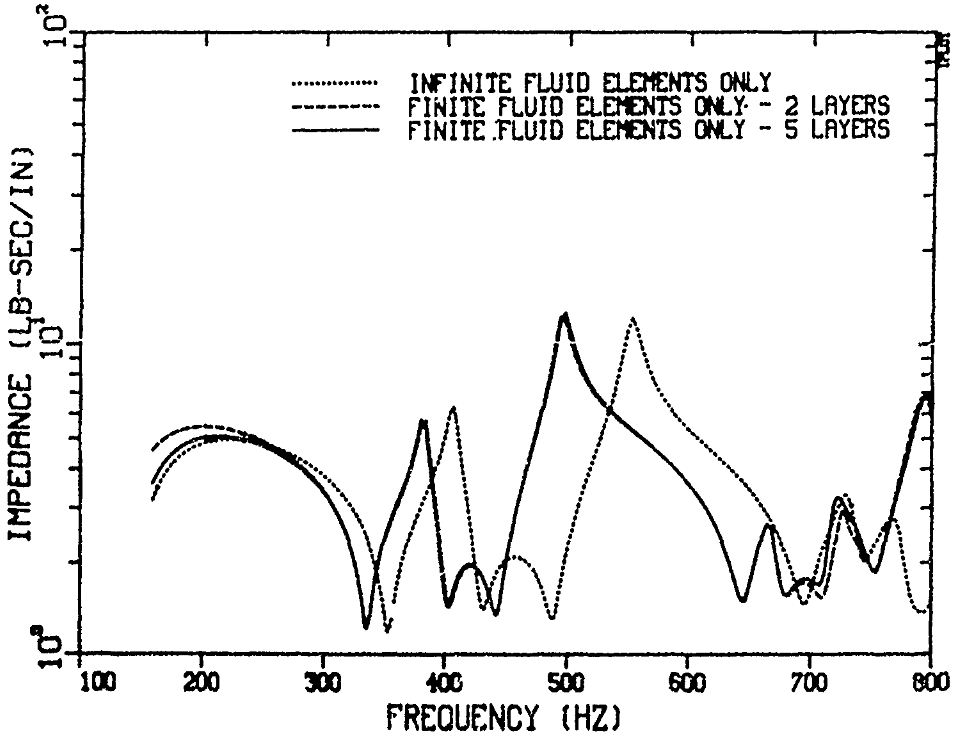

Fig. 4. Frequency response, comparisons using infinite elements only with two layers of finite elements

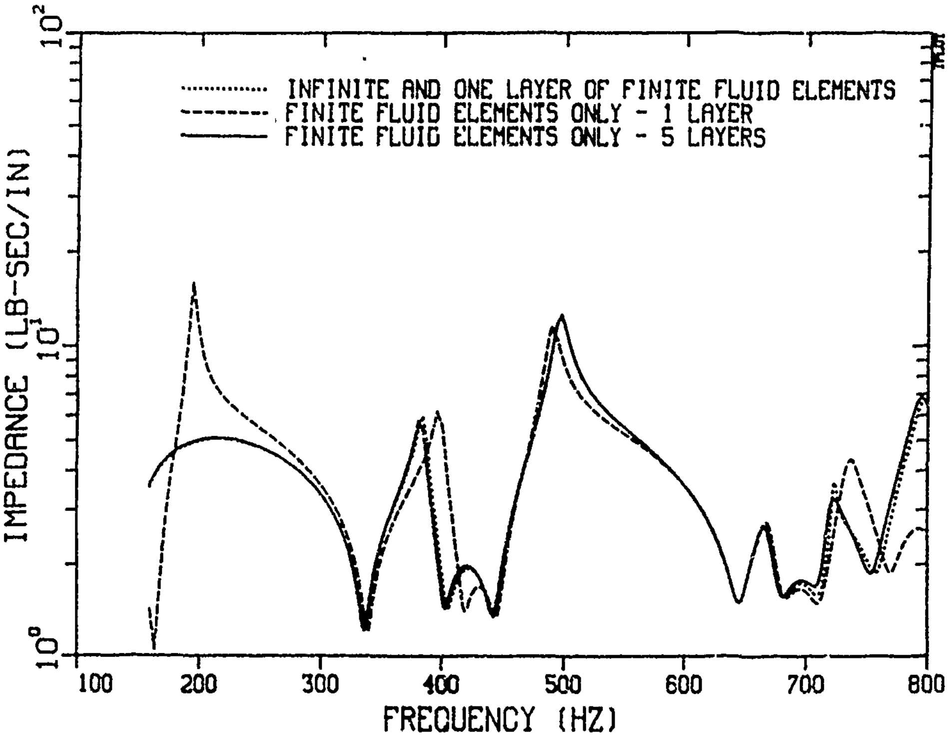

Fig. 5. Frequency response, comparisons using infinite elements over one layer of finite elements with one layer of finite elements.

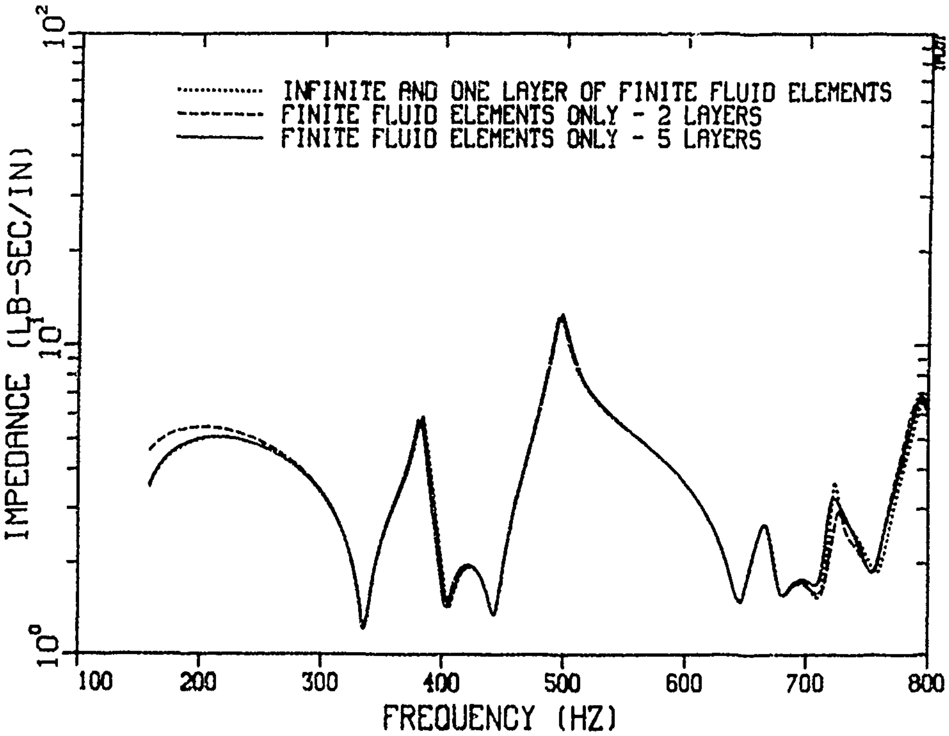

Fig. 6. Frequency response, comparisons using infinite elements over one layer of finite elements with two layers of finite elements.

It is expected that infinite elements hold more promise for applications to the Helmholtz equation problem. It is known that a finite fluid element model for the Helmholtz equation case must include six to eight layers of finite elements to follow the shape of the wave and must include damping boundary conditions on the outer boundary to account for energy dissipation by outgoing sound waves.[15] Infinite elements developed for the Helmholtz equation problem will include wave characteristics of the sound field in their shape functions and are expected to account for energy dissipation. The results obtained with the added mass elements show promise that the Helmholtz equation elements will be able to produce accurate solutions with the use of one layer of finite fluid elements. If this promise is realized, the use of infinite fluid elements will offer a considerable advantage over the use of only finite fluid elements. The infinite elements for the Helmholtz equation must also compete with the boundary integral equation method This method produces accurate solutions, but it is also expensive, since it requires the solution of systems of linear equations that are not banded and not symmetric. Infinite fluid elements offer the possibility that for some problems, the accuracy needed may be obtained at less expense.

APPENDIX A. VERIFICATION PROBLEM

To evaluate the performance of the infinite elements for representing the Laplace operator, the elements were used to solve the boundary value problem for the potential ϕ

$$\nabla^2\phi = 0$$

in the region outside of a bounded surface with boundary conditions $\phi = f$ prescribed on the surface. The boundary of the exterior region was a cylindrical surface with finite length and closed ends. The cylindrical boundary has length = 50 in. and radius = 10 in.

Symmetries in the problem reduced the number of elements needed to model the cylindrical boundary and the exterior region. A plane of symmetry midway between the ends and perpendicular to the axis of the cylindrical boundary permitted modeling only the half of the length of the cylindrical boundary and the half of the region on one side of the plane. Two additional planes of symmetry, mutually perpendicular and each containing the axis of the cylindrical boundary, permitted modeling one-fourth of the remaining cylindrical boundary and exterior region. Therefore, only one-eighth of the cylindrical boundary and exterior region needed to be modeled.

Figure 7 gives end and side views of the cylinder and the surrounding acoustic fluid. Three varieties of infinite elements were used to model the acoustic fluid in this problem. The infinite elements on the sides and on the end, except the circle of elements on the axis of the cylinder, are the eight-node infinite elements described earlier in this report. In the corner region, between the side and end sections of elements, the infinite elements have been modified to extend to infinity in two directions, r and z. The circle of infinite elements on the axis of the cylinder at the end of the cylindrical shell are similar to the eight-node elements, but their bases are triangles.

The infinite elements were evaluated by comparing the accuracy of a solution obtained using a model that incorporated infinite elements with the accuracy obtained using models that contained only finite elements. The three finite element models contained two, three, or four layers of finite elements. Since the potential becomes zero at large radii, the boundary condition $\phi = 0$ was assigned at the outer boundary of the finite element model. For the infinite element solution, the model of the exterior region consisted of one

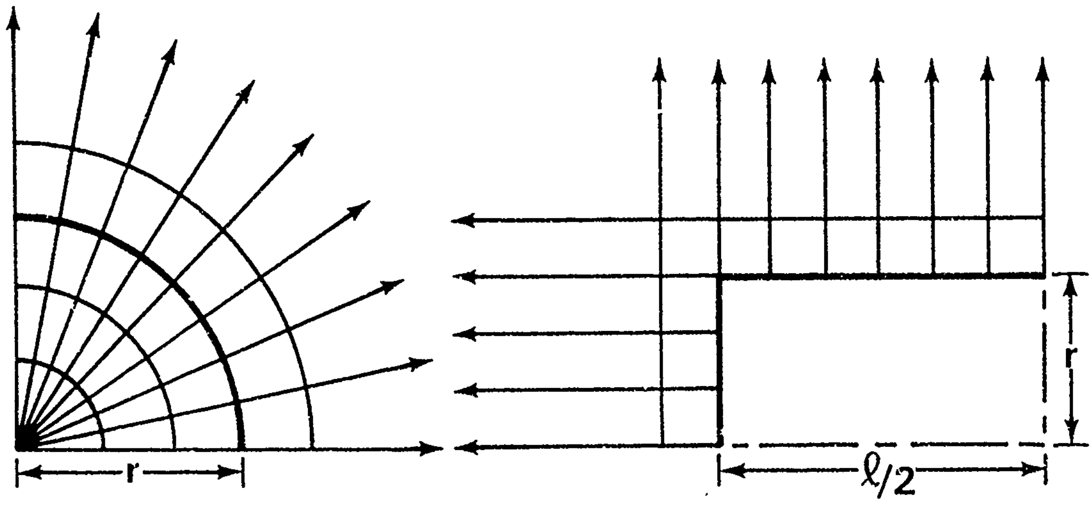

Fig. 7. Finite and infinite element model of the region surrounding the cylindrical boundary.

layer of finite Laplace elements over the cylindrical surface with infinite elements placed over this layer. This evaluation consists of comparisons of values obtained for the exterior potential at nodes on the surface between the finite and infinite elements. The standard for comparisons was a solution obtained using the boundary integral equation program BEASY. The boundary conditions were $\phi(\theta,z) = (1+z)\cos\theta$ on the cylindrical surface and $\phi(\theta,r) = 2.6\ r\cos\theta$ on the end. To be consistent with the boundary elements used with BEASY, the boundary conditions were computed at $\theta = 0°$, $45°$, and $90°$ and linearly interpolated between. Table 2 gives potentials computed at several points, the potentials for each point in the table have been normalized by dividing by the standard potential computed at the point.

TABLE 2. Potentials computed outside the cylindrical shell.

Point Coordinates			Potentials(normalized)				
			Methods				
r	θ	z	BIE	IE	FE4	FE3	FE2
6.67	22.5	29.17	1.000	0.955	1.050	1.044	1.027
10.0	45.0	29.17	1.000	0.914	0.875	0.861	0.820
13.14	56.25	20.83	1.000	0.999	1.002	0.979	0.917
13.14	56.25	8.33	1.000	0.990	0.966	0.927	0.840

Note: In Table 2, the acoustic fluid is modeled by
BIE - boundary integral equation program BEASY
IE - infinite elements over one layer of finite elements
FEi - finite elements only, i layers

APPENDIX B. EVALUATION PROBLEMS

Two test problems were used to evaluate the use of infinite elements for applications to fluid-structure interaction problems.

The first test problem is to compute the natural frequencies of a submerged cantilevered plate This plate has the properties of the first of two plates analyzed by Marcus,[13] and the problem corresponds to his deeply submerged case. The length-to-chord ratio of the plate equals 2.0 and the thickness-to-chord ratio equals 0.0131. The plate has the material properties of steel, with Young's modulus = 3 0 10^7 lb/in^2, Poisson's ratio = 0.3, and mass density = 7.324 10^{-4} lb- sec^2/in^4. The fluid is assumed to have mass density = 9.34 10^{-5} lb- sec^2/in^4 and sound speed = 6000 in/sec. The frequencies computed here compare well with those computed by Marcus. However, his formulation is slightly different from the formulation used here, since he included the fluid matrix Q that derives from the $\partial^2 p/\partial t^2$ term (see Eq. 2), so his results are not compared with the results from this work. Rather, comparisons were made using a standard consistent with the formulation of the added mass approximation used in this work.

By using boundary conditions that represent the line of symmetry running lengthwise along the center of the plate, only one-half of the plate in the chordwise direction need be modeled. The finite element model of the plate consists of five NASTRAN QUAD1 plate elements along the length and three elements across the width. One end of the plate is constrained to represent the clamped end condition. The fluid is modeled with NASTRAN three-dimensional IHEX1 isoparametric finite elements and with infinite elements. The fluid elements extend for two elements outside of the plate in the directions of the length and the width. By using half of the mass and bending stiffness of the plate, only the fluid on one side need be explicitly modeled. On the outer boundary of the fluid finite element model the boundary condition p = 0 is applied. Figure 8 shows the finite element model of the plate and the acoustic fluid.

The finite element solution of the submerged plate problem is implemented using the NASTRAN program in three runs. The first NASTRAN run computes a force vector using a unit pressure load on the fluid-structure interface. The resulting pressure load vector is processed to produce the area matrix A, and in the second NASTRAN run the matrix A is used to form the added mass matrix. The third NASTRAN run

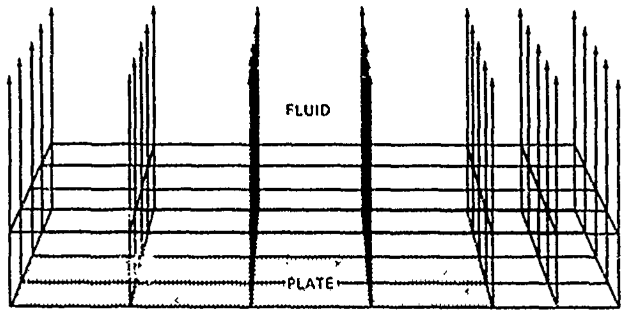

Fig. 8. Finite element model of the submerged plate and surrounding fluid.

adds the added mass matrix to the mass matrix of the structure and computes the natural frequencies of the fluid-structure system.

Several configurations of fluid elements were used to compute the natural frequencies. Since Marcus[13] showed that for low frequencies the natural frequencies of a submerged plate can be computed accurately using only a few layers of finite elements (he got good accuracy using three layers), a set of natural frequencies was computed using five layers of finite fluid elements and this set was used as the standard for comparison. Comparisons with frequencies obtained using three and four layers showed that the conservative model with five layers was well converged. Table 3 shows frequencies obtained in the test.

The second test problem is to compute the forced frequency response of a submerged cylindrical shell with closed ends. The amplitude of the response is computed at a transfer point on the cylinder for a range of frequencies. The response is due to a harmonic excitation of unit amplitude at a separate drive point

TABLE 3. Natural frequencies of a submerged plate.

Layers of Finite Elements	Infinite Elements Used	Frequencies(Hz) Mode 1	2	3	4	5
5	No	5.04	28.7	32.9	94.9	95.5
0	Yes	5.26	29.4	34.0	96.4	97.4
1	Yes	5.16	29.4	33.6	95.5	96.9
2	No	5.30	29.4	34.1	97.5	98.9

The finite element model for the surrounding fluid, in which infinite elements are placed over one layer of finite elements, had the same configuration as that used for the exterior region in the Dirichlet problem in Appendix A (see Fig. 7). As in the Dirichlet problem, symmetries reduced the number of elements needed to model the cylinder and the surrounding fluid, so that only one-eighth of the cylinder and surrounding fluid needed to be modeled. The finite element analysis for this problem is also implemented using the NASTRAN program. The matrices for the infinite elements and the area matrix are generated by a separate program and entered during the NASTRAN run on DMIG cards images. As in the submerged plate problem, the structure (that is, the cylindrical shell) was modeled with NASTRAN QUAD1 plate elements and the fluid was modeled with NASTRAN three-dimensional HEX1 isoparametric finite elements. Figures 3-6 show frequency response curves for the submerged cylindrical shell.

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